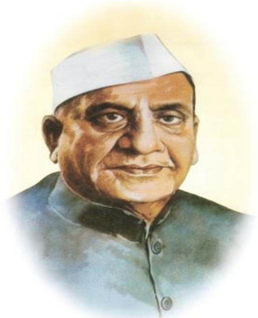

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PERMUTATION GROUP

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CONTENTS

GROUPS

- **Definition**:-An algebraic structure $(G, *)$where G is a non-empty set with the binary operation $(*)$defined on it is said to be group if following axioms are satisfied.

1] G1: Closure property

$a*b \in G$ for all $a, b \in G$

2] G2: Associative property

$a*(b*c)=(a*b)*c$ for all $a, b, c \in G$

3] G3: Existence of identity

There exist an element $e \in G$ such that

$e*a=a*e=a$ for all $a \in G$

Therefore e is called identity element of G

4] G4:Existance of inverse

For each $a \in G$ there exist $b \in G$ such that ,

$a*b=b*a=e$

Then element b is inverse of a.

PERMUTATION GROUP

- Definition:-

Let S be a finite set having n distinct elements . A one-one mapping S to S itself is called a permutation of degree n on set S.

Symbol of permutation :

$$Let\ S = \{a_1, a_2, a_3 ....a_n\} be\, a\ finite\ set$$
$$with\, n\, distinct\ elements.let\ f : S \rightarrow S\, be\, a\, 1-1 mapping$$
$$of\ S\, on\, to\, itself\ .$$
$$f(a_1) = b_1, f(a_2) = b_2 ....f(a_n) = b_n,\ then\, written\ as\ follows$$
$$f = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 ....a_n \\ b_1 & b_2 & b_3 & b_4 ....b_n \end{pmatrix}$$

Degree of permutation

The number of elements in a finite set S is called as degree on permutation. If n is a degree of permutation mean having n! permutations

Example: Let S=(1,2,3,4,5)and f is a permutation on set S itself.

5! = 120 permutations

Identity Permutation

If I is a permutation of degree n such that I replaces each element by itself then I is called identity permutation of degree n.

i.e. f(a)=a

Ex. $I = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}$

$\therefore$ I is identity permutation.

Identity permutation is always even.

EQUALITY OF TWO PERMUTATIONS

Two permutations f and g with degree n are said to be equal if f(a)=g(a).

Ex. $f=\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$ $g=\begin{pmatrix} 4 & 3 & 2 & 1 \\ 2 & 4 & 1 & 3 \end{pmatrix}$

$\therefore$ f(a)=g(a)

Product of two permutations

The product or composition of two permutation f and g with degree n denoted by f. g, obtained by first carrying out operation defined by f and then g.

i.e. f . g(x) = f(g(x))

Ex. $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 1 & 5 & 3 \end{pmatrix}$ $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 2 & 3 & 1 \end{pmatrix}$

$$f.g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 1 & 5 & 3 \end{pmatrix} . \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 2 & 3 & 1 \end{pmatrix}$$

$$f.g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 1 & 2 \end{pmatrix}$$

$$g.f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 2 & 3 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 1 & 5 & 3 \end{pmatrix}$$

$$g.f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 2 & 1 & 4 \end{pmatrix}$$

$$\Rightarrow \quad f.g \neq g.f$$

CYCLE

- Definition:-

  Let S be a finite set having n distinct elements. A permutation f of degree n on set S , is called as cycle of length k iff there exist k is distinct element.

$$a_1, a_2, a_3 .... a_k \qquad (k \leq n)$$

Example: $f(a_1) = a_2, f(a_2) = a_3 ..... f(a_{k-1}) = a_k, f(a_k) = a_1$

$$cycle = (1,2,3,4,5)$$

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \end{pmatrix}$$

If the length of cycle is k , then there are k-1 transpositions

TRANSPOSITION

A cycle of length two is called as transposition.

Every permutation can be expressed as product of transpositions

i.e. cycle of permutation

(a1 , a2, a3, ……………….., an)

=(a1,an),(a1,an-1),……………,(a1,a3),(a1,a2)

These are the transpositions.

EVEN PERMUTATION

If the number of transposition is even then permutation is even.

Example:-

a*)(1,2)(1,3)(1,4)(2,5)*

Given permutation is (1,2)(1,3)(1,4)(2,5)

$\therefore$ Number of transposition= 4 =even number.

Hence the given permutation is an even permutation.

- Inverse of even permutation is even.

ODD PERMUTATION

If the number of transposition is odd then permutation is odd.

Example:-

a*) (1,2,3,4,5)(1,2,3)(4,5)*

Given permutation is (1,2)(1,3)(1,4)(1,5)(1,2)(1,3)(4,5)

∴ Number of transposition= 7 =odd number.

Hence the given permutation is an odd permutation.

- Inverse of odd permutation is odd.

- **Product of two even permutation is an even permutation.**

f = (1 2 3) is an even permutation

f.f = (1 2 3) (1 2 3) = (1 3 2) = (1 2) (1 3) = even permutation

- **Product of two odd permutation is an even permutation**

f1 = (1 2) and f2 = (1 3)

F1.f2 = (1 3 2) = (1 2) (1 3)

= even permutation

- **Product of odd and even permutation is an odd permutation**

f1 = (1 2 3) and f2 = (2 3)

F1.f2 = (1 2 3) (2 3)

= (1 3) (1 2) (2 3)

= odd permutation

THANK YOU